**Chapter 02: First Order, First Degree, Ordinary Differential Equations**

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General Form:

Let

This can be generally written as . This is another general form of a first order, first degree, ordinary differential equation.

Here, , and are all functions of either , , and or constants.

Solving Methods:

1. Separation of Variables
2. Homogenous Differential Equations
3. Linear Differential Equations
4. Exact Differential Equations

## Chapter 2.1: Exact Ordinary Differential Equation

### Total Differential

– This is the partial derivative of with respect to .

– This is the partial derivative of with respect to .

– This is the total differential of .

The first order first degree ordinary differential equation is called an exact differential equation, if the left-hand side is the total differential of some function .

In other words, the expression is called an exact differential if there exists some for which this expression is the total differential of .

– This is the solution of the exact differential equation.

## Chapter 2.2: Separable Equations and Equations Reducible to This Form

### Separation of Variables

If is a function of only and is a function of only , then the variables can be said to be separated.

The equation can now be integrated to find the solution.

### Reducible to Separable

Let

Let

Find the particular solution of , where .

Exercise

so, this equation is not exact.

This is a function of only.

Integrating factor =

Thus, the equation is now exact.

## Chapter 2.3: Linear Equations and Bernoulli Equations

### Linear Differential Equation of Order One

Standard Form: where and are functions of and constant.

Solving Method:

This equation is not exact.

Let be the integrating factor.

This is an exact equation.

This is a separable differential equation.

Integrating factor =

This is a linear differential equation.

Integrating factors

(The integration is simplified as )

Let .

where and

### Bernoulli Differential Equation (Reducible to Linear)

Standard Form:

Let

This is a linear equation.

Let .

Let

### Application of 1st Order 1st Degree Ordinary Differential Equations

The population of a town was 60,000 in 1990 and had increased to 63,000 by 2000. Assuming the population is increasing at a rate proportional to its size at any time, estimate the population in 2010.

Let the size of the population be at any time.

A patient is receiving drug treatments. When measured, there is 0.5 mg of the drug per liter of blood. After 4 hours, there is only 0.1 mg per liter of blood. Assume that the amount of drugs in the blood at time . Find how long it takes for there to be only 0.05 mg of the drug per liter of blood.

Let the amount of drug at any time be .

A circuit consisting of resistance ohms and an inductance is connected to a battery of constant voltage . Find the current in at time after the circuit is closed.

An inductance and resistance each cause a drop in voltage. The drop due to resistance is and the ddrop due to inductance is . Voltage supplied by the battery is equal to the voltage drop.

This is a linear differential equation.

A stone weighing 4 lb falls from rest towards the Earth from a great height. As it falls, it is acted upon by air resistance that is numerically equal to .

a. Find the velocity and distance fallen at time .

b. Find the velocity and distance fallen at .

Exercise:

This is an exact equation.

### Exact Differential Equation

If is exact,

Proof:

If , then we show that is exact. This means that we must prove that there exists a function such that

Let us assume that satisfies .

Total derivative:

Exercise:

Thus, the equation is an exact differential equation.

Exercise:

Find the family of curves that satisfy the differential equation

Exercise

Since , the differential equation is exact.

### Grouping Method

* Integrate each term.
* Group terms that are the same.

Exercise:

Exercise:

### Reducible to Exact

Integrating Factor: If is not exact, but can be made exact by multiplying with another inexact equation, this inexact equation is called the integrating factor.

Exercise:

Thus, the differential equation is not exact.

Multiplying both sides by ,

Thus, the integrating factor is .

### Rules of finding the integrating factor

* If , a function of only , integrating factor
* If , a function of only , integrating factor
* If and the equation is homogenous, integrating factor
* If , integrating factor

Find the equation of the curve passing through the point and whose differential equation is

### Homogenous Differential Equations

Homogenous Function: If is a function of two variables and and if can be expressed by or , then is called a homogenous function of degree .

This function can be expressed in 3 ways:

All 3 of these prove that is a homogenous function.

The first order, first degree differential equation is homogenous if and are both homogenous equations of the same degree.

Thus, the degree of must be .

### Method of solving homogenous differential equations

Let .

Separate the variables, integrate and replace .

Find the particular solution of ,

This is a homogenous differential equation.

Let .

Let .

Here, solving with is complicated, so is used instead.

Let .